SECOND DEGREE (PREMIÈRE L)

What is the problem posed by the Babylonians?

How are called the equations involved in this kind of problem? Explain why.

What is the general form of such equations?

Describe the method the Babylonians stated in the Ancient Times.

Now let's find the answer by using a modern strategy. Let x be the unknown side (the **width** or breadth of the field). Make a drawing.

Express the area of the field in terms of x. Check that the problem corresponds with solving $x^2 + 6x - 55 = 0$.

Input the function $f(x) = x^2 + 6x - 55$ in your calculator, and then determine the answer of the equation $x^2 + 6x - 55 = 0$ graphically.

AL – KHWARIZMI AND QUADRATICS

Explain the context that allowed the improvement of mathematics from the VIIth century in the Arab Empire.

Who was Al- Khwarizmi ?

A page from al-Khwārizmī's Algebra

What are his two main contributions to Mathematics?

1-

2-

Why is Al- Khwarizmi's algebra so important for our modern Mathematics?

AL- KHWARIZMI'S METHOD FOR SOLVING QUADRATIC EQUATION

His quadratic equations are composed of units, roots and squares. For example, to Al-Khwarizmi a unit was a number, a root was x, and a square was x^2. *Note that Al-Khwarizmi's mathematics is done entirely in words with no symbols being used.*

He first reduces an equation $ax^2 + bx + c = 0$ to one of six standard forms:

- squares equal roots ($ax^2 = bx$)
- squares equal number ($ax^2 = c$)
- roots equal number ($bx = c$)
- squares and roots equal number ($ax^2 + bx = c$)
- squares and number equal roots ($ax^2 + c = bx$)
- roots and number equal squares ($bx + c = ax^2$)

The reduction is carried out using the two operations of **al-jabr** (which turned into "algebra") and **al-muqabala**.

- "**al-jabr**" means "completion". It is the process of removing negative terms from an equation as .

For example, using one of al-Khwarizmi's own examples, "al-jabr" transforms:

$$x^2 = 40\,x - 4\,x^2 \text{ into } 5\,x^2 = 40\,x$$

- "**al-muqabala**" means "balancing" and is the process of reducing positive terms of the same power when they occur on both sides of an equation.

For example, two applications of "al-muqabala" reduces :

$$50 + 3\,x + x^2 = 29 + 10\,x \text{ to } 21 + x^2 = 7\,x$$

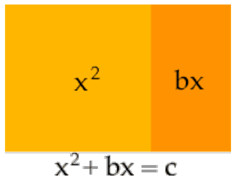
Task 3 : Use Al-Khwarizmi's operation to transform $2x^2 + 8x + 4 = 3x^2 - 5x + 1$ into the 6th standard form ($bx + c = ax^2$).

Al-Khwarizmi described the rule used for solving each type of quadratic equation and then presented a proof for each example. He used both algebraic methods of solution and geometric methods.

Let's consider the 4th form of the previous list: $\boldsymbol{ax^2 + bx = c}$

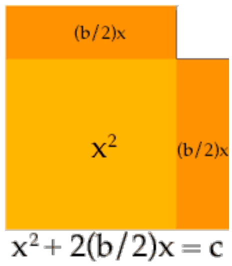
Algebraic method: For example, to solve the equation $x^2 + 10x = 39$ Al-Khwarizmi writes :

"... a square and 10 roots are equal to 39 units. The question therefore in this type of equation is about as follows: what is the square, which combined with ten of its roots, will give a sum total of 39? The manner of solving this type of equation is to take one-half of the roots just mentioned. Now the roots in the problem before us are 10. Therefore take 5, which multiplied by itself gives 25, an amount which you add to 39 giving 64. Having taken then the square root of this, which is 8, subtract from it half the roots, 5 leaving 3. The number three therefore represents one root of this square, which itself, of course is 9. Nine therefore gives the square. "



Task 4: Geometric method: c is the area of the whole rectangle.

1. Explain the link between the equation and the areas.



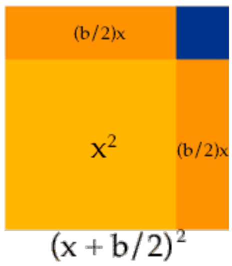
Al Khwarizmi chops the bx term in half, resulting in two rectangles of area $\frac{b}{2}x$, which he then rearranges along the edges of the square of side x:

2. What is the formula to compute areas of rectangles?

$\mathscr{A} =$

3. The area of one rectangle is $\frac{b}{2}x$, meanwhile its length is x, so the width of the rectangle is ......

4. If we add a little square, top right in the diagram, we can make one big square. The area of the little blue square is $(\frac{b}{2})^2$. Explain why.



5. Find the side of the big square in terms of x.

6. Deduce why the area of the big square is $(x+\frac{b}{2})^2$.

7. The area of the big square is also equal to *(the little blue square + c),* therefore

$c+(\frac{b}{2})^2=(x+\frac{b}{2})^2$

thus $\sqrt{c+(\frac{b}{2})^2}=x+\frac{b}{2}$

thus $x=\sqrt{c+(\frac{b}{2})^2}-\frac{b}{2}$

8. Find a solution of $x^2 + 10x = 39$ using the previous formula.

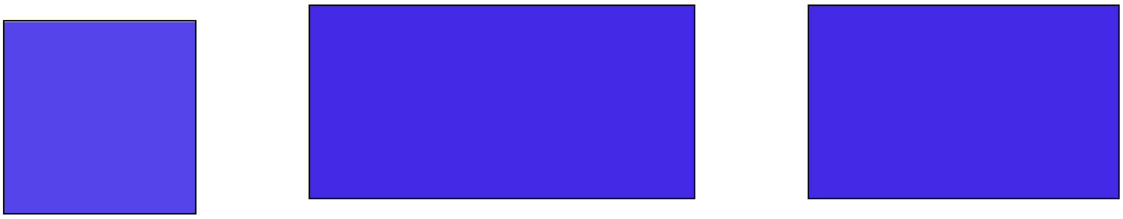
We have $c =$ …. and $b =$ ……

Thus x =

9. Compare with the Babylonians' method. What is Al-Khwarizmi's bringing-in ?

Task 5 :

Among the following rectangles, circle the one you think is the most attractive and well-balanced :



1 2 3

Give the reasons of your choice.

Now, measure each rectangle's length and width, and compare the ratio of length to width for each rectangle:

Rectangle	1	2	3
Length (L) cm			
Width (w) cm			
$\frac{L}{w}$			

Task 6 :

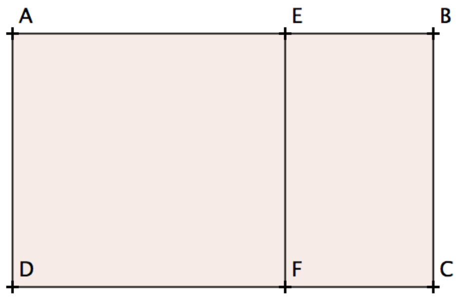
Find at least three rectangles in the room. Measure their dimensions. Fill in the following table :

Rectangle			
Length (L) cm			
Width (w) cm			
$\frac{L}{w}$			

Which one seems to be more attractive?

Task 7 (première L) :

We consider a rectangle ABCD of **length** AB=x (given $1<x<2$) and **width** AD=1. Let E be the point of [AB] and F the point of [DC] such that AEFD is a square



For each rectangle (ABCD and BEFC), find the ratio of the length to the width in terms of x.

r_{ABCD}=

r_{BEFC}=

ABCD is said to be a **golden rectangle** if $\mathbf{r_{ABCD}=r_{BEFC}}$.

Write the corresponding equation in terms of x.

One of the solutions of this equation is :

$$\phi = \frac{1+\sqrt{5}}{2}$$

This number is called the golden ratio or divine proportion.

Give an approximate value for ϕ. (Round off to the nearest thousandths).

$$\phi \cong$$

Task 8: Write a presentation about the use of the golden ratio in Architecture.

Task 9: Write a presentation about the use of the golden ratio in Arts.

Task 10: Write a presentation about the use of the golden ratio in Nature.

Task 11: Being stuck on a Architect …

You're an architect and you have to draw the planes of a construction. As you're definitely a great artist, you decide to use the magnificent and unforgettable GOLDEN RECTANGLE!

But can you draw it? Explain the protocol of construction.