SUBJECT 4 : THE MOON

Task 1: Distance Moon-Earth

In 1751, the astronomers Lalande and Lacaille both measured the distance Moon-Earth from Berlin (B) and Cape Town (C). The zenith angle of the Moon (M) seen from Berlin is $\widehat{bBM}$ =31.77°. The zenith angle of the Moon seen from Cape Town is $\widehat{cCM}$ =56.06°. Work out the distance Moon-Earth ME.

[redacted]

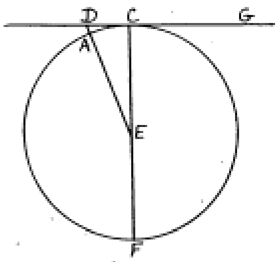
Reminder: The latitude of Berlin is 52.2°N, the latitude of Cape town is 33.92°S. The radius of the Earth is approximately 6370 km.

Task 2: Below is an extract of Galileo's *Sidereus Nuncius* (1610).

One of Galileo's drawings of the moon. 1610 A.D.

CALCULATION TO SHOW THAT THE HEIGHT OF SOME LUNAR MOUNTAINS EXCEEDS FOUR ITALIAN MILES

I think that it has been sufficiently made clear, from the explanation of phenomena which have been given, that the brighter part of the Moon's surface is dotted everywhere with protuberances and cavities; it only remains for me to speak about their size, and to show that the ruggednesses of the Earth's surface are far smaller than those of the Moon's; smaller, I mean, absolutely, so to say, and not only smaller in proportion to the size of the orbs on which they are. And this is plainly shown thus:—As I often observed in various positions of the Moon with reference to the Sun, that some summits within the portion of the Moon in shadow appeared illumined, although at some distance from the boundary of the light (the terminator), by comparing their distance with the complete diameter of the Moon, I learnt that it sometimes exceeded the one-twentieth (1/20th) part of the diameter. Suppose the distance to be exactly 1/20th part of the diameter, and let the diagram represent the Moon's orb, of which CAF is a great circle, E its centre, and CF a diameter, which consequently bears to the diameter of the Earth the ratio 2:7; and since the diameter of the Earth, according to the most exact observations, contains 7000 Italian miles. CF will be 2000, and CE 1000; and the 1/20th part of the whole CF, 100 miles. Also let CF be a diameter of the great circle which divides the bright part of the Moon from the dark part (for, owing to the very great distance of the Sun from the Moon this circle does not differ sensibly from a great one), and let the distance of A from the point C be 1/20th part of that diameter; let the radius EA be drawn, and let it be produced to cut the tangent line GCD, which represents the ray that illumines the summit, in the point D.



Then the arc CA or the straight line CD will be 100 of such units, as CE contains 1,000. The sum of the squares of DC, CE is therefore 1,010,000, and the square of DE is equal to this; therefore the whole ED will be more than 1,004; and AD will be more than 4 of such units, as CE contained 1,000. Therefore the height of AD in the Moon, which represents a summit reaching up to the Sun's ray, GCD, and separated from the extremity C by the distance CD, is more than 4 Italian miles; but in the Earth there are no mountains which reach to the perpendicular height even of one mile. We are therefore left to conclude that it is clear that the prominences of the Moon are loftier than those of the Earth.

1. Read the text carefully: what is the purpose of this extract?
2. Translate the title of Galileo's book in English.
3. Use the letter-labels to fill in the blanks below.
   a. The letter ___ indicates the Moon's centre.
   b. Segments ___, ___ and___ are each a radius of the Moon.
   c. Segment ___ represents a mountain whose top is at point D.
   d. Segment ___ indicates the beam of sunlight that touches, at point C, the boundary between the Moon's light and dark halves – technically, the terminator– and illuminates the mountain.
4. Galileo found a right triangle of interest: triangle ECD. Again, use the letter-labels in Galileo's diagram to fill in the blanks below regarding statements about triangle ECD.
   a. Angle ___ is the right angle.
   b. Side ___ is a lunar radius, about _______ miles.
   c. Side ___ is the distance from the terminator to the illuminated mountain peak, as seen from Earth.
   d. Side ___ is the triangle's hypotenuse.
   e. This hypotenuse encompasses the sum of two segments: another lunar radius, segment ___, plus the height of the mountain, segment ___.

In *The Starry Messenger*, Galileo wrote that the illuminated peaks appeared as far as $\frac{1}{20}$ of the Moon's diameter from the terminator.

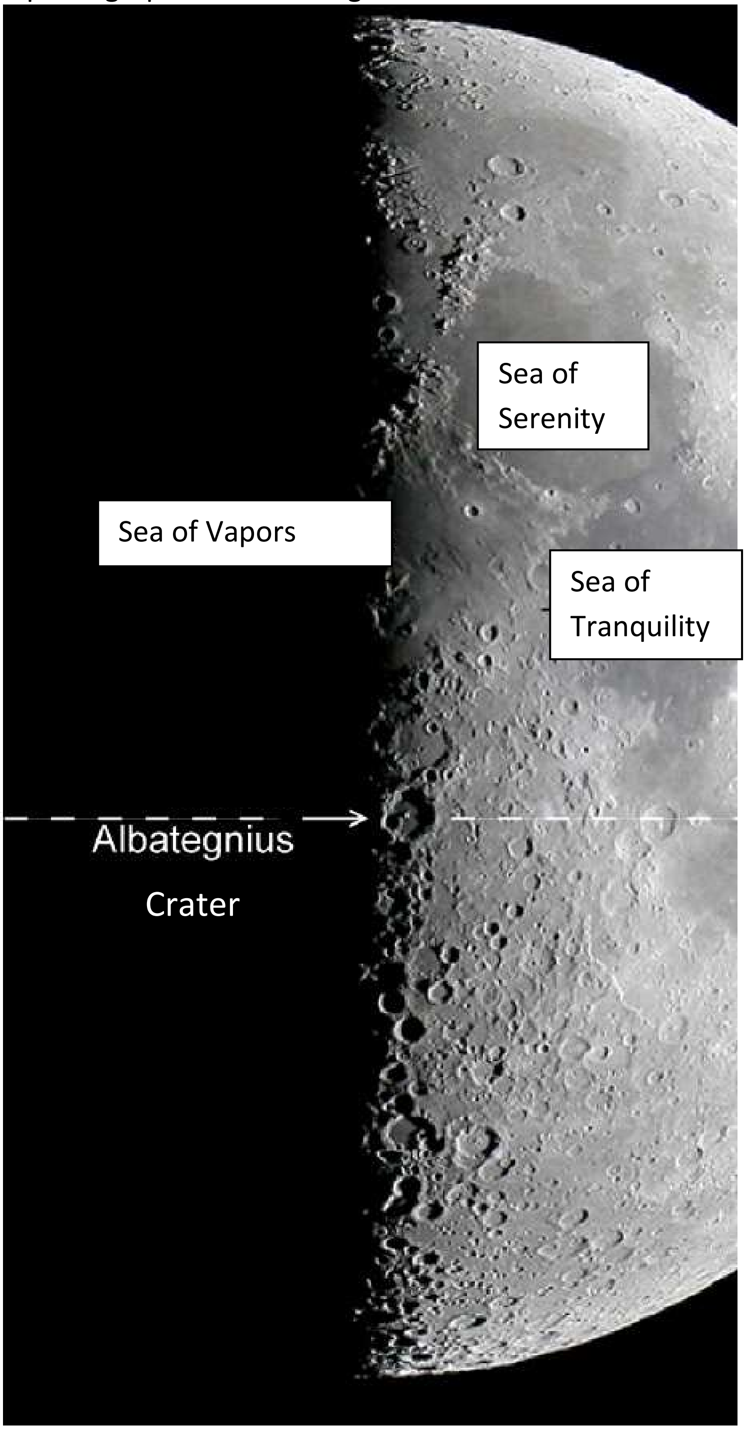
5. Use the diameter of the Moon to work out how far into the Moon's dark side these illuminated peaks were situated, according to Galileo.
6. To which side of the right triangle ECD does this answer correspond?
7. Use the Pythagorean Theorem for the hypotenuse of triangle ECD.
8. Deduce the height of the mountain itself. In fact, your answer represents the minimum height of lunar mountains; they must be at least this tall to poke up into the sunlight.
9. A more accurate value for the diameter of the Moon is 2160 miles. Find the height of the mountain.
10. Given a mile is 1685 metres, compare the height of lunar mountains to that of mountains on Earth. Your answer must include a direct number comparison.

Task 4
Assume the Moon is spherical with radius 1740 km. You've just landed in the Sea of Showers and at the horizon you see the Mons Blanc (3,600 m high). How far are you from this peak?

In this problem, we'll disregard the observer's height.

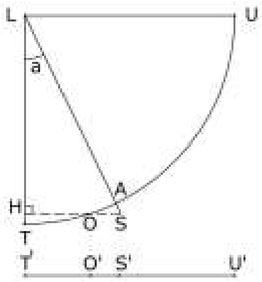
Task 5

A photograph of the waxing crescent Moon is shown below.



1. Estimate the diameter of the Albategnius crater knowing the diameter of the Moon is approximately 3,500 km.

2. The figure below shows a cross-section of the Moon following the dotted line on the photo above.



A: bottom of the rim of the crater
S: top of the rim of the crater
O: endpoint of the shadow of the rim of the crater
T: point of the Moon's terminator
L: centre of the Moon

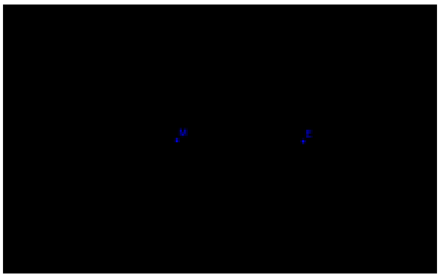
Find an estimate of the height of the rim of the crater.

Task 6 : the Lunar month

Sidereal period of the Moon

The Moon went by the star Aldebaran on 1996 November 25 at 16:00, then again on 1996 December 23 at 0:00. Calculate the time interval between the two events: this is the sidereal period of the Moon.

The Moon about the Earth



Assume the Earth is a fixed point. The Moon makes a complete orbit around the Earth with respect to the fixed stars about once every 27.3 days (its sidereal period).
At the time $x=0$, the Earth, the Moon and the Star Aldebaran are collinear. Let's note x the number of days since this time and $a(x)$ the number of rotations about the Earth.

Complete the table

nr of days x					54.6	1	40	x
nr of rotations b(x)	1	3	0.5	0.1				

Give the expression of the function a in terms of x.

The Earth about the Sun

While the Moon turns about the Earth, the Moon-Earth system revolves around the Sun. We consider the Earth's orbit is circular and run in 365 days at constant speed. Assume the Earth, the Sun and the star Aldebaran are collinear in that order when $x=0$.
Let $b(x)$ be the number of turns of the Earth about the Sun.

Complete the table

nr of days x				1	40	x
nr of rotations $a(x)$	1	2	0,1			

Give the expression of the function b in terms of x.

Composition of both motions

When $x=0$, the position of the Earth is E_0, and the Moon is at M_0, between the Earth and the Sun. We can see its dark face: it is the new moon.

The next new moon happens when the Sun, the Moon and the Earth are collinear again (S, M and E) as shown on the diagram.

The aim of this problem is to calculate a lunar month, i.e. the number of days x that passed between those two new moons.

As Aldebaran is far from the solar system, we'll consider the lines (E_0z_0) and (Ez) are parallel.

1. Compare the angles $\widehat{E_0SE}$ and $\widehat{zES}$.
2. Explain why we must get $a(x)=1+b(x)$.
3. Deduce the number of days x between the two new Moons.