SECOND DEGREE (PREMIÈRE S)

What is the problem posed by the Babylonians?

How are called the equations involved in this kind of problem? Explain why.

What is the general form of such equations?

Describe the method the Babylonians stated in the Ancient Times. Explain their reasoning.

Task 1

Now let's find the solution by using a modern strategy. Let x be the unknown side (the **width** of the field). Express the area of the field in terms of x. Check that the problem corresponds with solving $x^2 + 6x - 55 = 0$.

Show that $x^2 + 6x - 55 = 0$ is equivalent to $(x-5)(x+11) = 0$. Deduce the answer.

If it exists, the **factorized** form of a **trinomial** of the second degree ax^2+bx+c, provided $a \neq 0$ is :

x_1 and x_2 (or x_0) are called **…………** of the **trinomial** ax^2+bx+c i.e. the trinomial **vanishes** as x takes those values.

The **completed square form** of a trinomial ax^2+bx+c is :

To solve a **quadratic** equation of the form $ax^2+bx+c=0$, provided $a \neq 0$:

Compute the ………………………… :

$\Delta =$

3 cases :

- **If** Δ ….0, **then** there is no solution in the set of real numbers.
- **If** Δ ….0, **then** there is one solution in the set of real numbers :

$x_0 =$

- **If** Δ …0, **then** there are two solutions in the set of real numbers :

$x_1 =$ and $x_2 =$

The solutions of the equation _if they exist_ are sometimes called **…………** of the **trinomial** ax^2+bx+c.

Task 2 

1. Write an algorithm that gives the discriminant and the solution of a quadratic equation.
2. Write the corresponding program on your calculator.
3. Write a program that gives the solutions of a second-degree equation (3 cases).
4. Solve $x^2+6x-55=0$ and check with your program. Try some other examples.

AL – KHWARIZMI AND QUADRATICS

Explain the context that allowed the improvement of mathematics from the VIIth century in the Arab Empire.

Who was Al- Khwarizmi ?

A page from al-Khwārizmī's Algebra

What are his two main contributions to Mathematics?

1-

2-

Why is Al- Khwarizmi's algebra so important for our modern Mathematics?

AL- KHWARIZMI'S METHOD FOR SOLVING QUADRATIC EQUATION

His quadratic equations are composed of units, roots and squares. For example, to Al-Khwarizmi a unit was a number, a root was x, and a square was x^2. *Note that Al-Khwarizmi's mathematics is done entirely in words with no symbols being used.*

He first reduces an equation $ax^2 + bx + c = 0$ to one of six standard forms:

- squares equal roots ($ax^2 = bx$)
- squares equal number ($ax^2 = c$)
- roots equal number ($bx = c$)
- squares and roots equal number ($ax^2 + bx = c$)
- squares and number equal roots ($ax^2 + c = bx$)
- roots and number equal squares ($bx + c = ax^2$)

The reduction is carried out using the two operations of **al-jabr** (which turned into "algebra") and **al-muqabala**.

- "**al-jabr**" means "completion". It is the process of removing negative terms from an equation as .

For example, using one of al-Khwarizmi's own examples, "al-jabr" transforms:

$$x^2 = 40\,x - 4\,x^2 \text{ into } 5\,x^2 = 40\,x$$

- "**al-muqabala**" means "balancing" and is the process of reducing positive terms of the same power when they occur on both sides of an equation.

For example, two applications of "al-muqabala" reduces :

$$50 + 3\,x + x^2 = 29 + 10\,x \text{ to } 21 + x^2 = 7\,x$$

Task 3 : Use Al-Khwarizmi's two operations to transform $2x^2 + 8x + 4 = 3x^2 - 5x + 1$ into the 6th standard form ($bx + c = ax^2$).

Al-Khwarizmi described the rule used for solving each type of quadratic equation and then presented a proof for each example. He used both algebraic methods of solution and geometric methods.

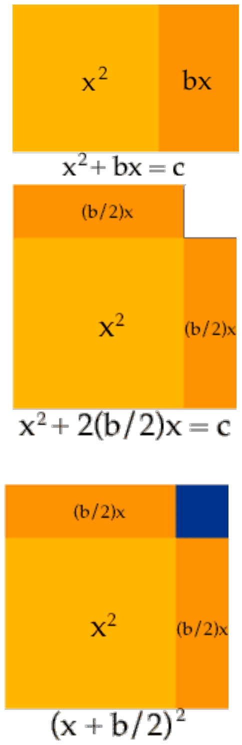
Let's consider the 4th form of the previous list: $ax^2 + bx = c$.

For example, we wish to solve the equation $x^2 + 10x = 39$.

<u>Algebraic method:</u>

Al-Khwarizmi writes :

"... a square and 10 roots are equal to 39 units. The question therefore in this type of equation is about as follows: what is the square, which combined with ten of its roots, will give a sum total of 39? The manner of solving this type of equation is to take one-half of the roots just mentioned. Now the roots in the problem before us are 10. Therefore take 5, which multiplied by itself gives 25, an amount which you add to 39 giving 64. Having taken then the square root of this, which is 8, subtract from it half the roots, 5 leaving 3. The number three therefore represents one root of this square, which itself, of course is 9. Nine therefore gives the square. "



Task 4: <u>Geometric method :</u> c is the area of the whole rectangle.

Al Khwarizmi chops the bx term in half, resulting in two rectangles of area $\frac{b}{2}x$, which he then rearranges along the edges of the square of side x :

1. Explain why the width of the rectangles is $\frac{b}{2}$.

2. If we add a little square, top right in the diagram, we can make one big square. What is the area of the little square?

3. Find two ways to express the area of the big square:

4. Deduce the solution of the equation.

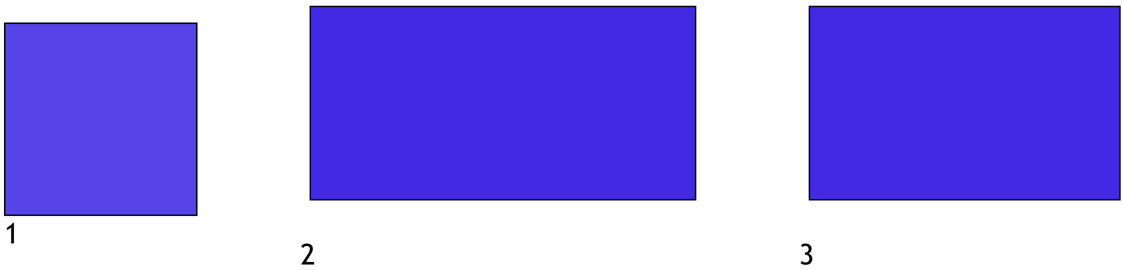
5. What is missing in his solution of $ax^2 + bx = c$?

6. Compare with the Babylonians' method. What is Al-Khwarizmi's bringing-in?

THE GOLDEN RATIO

Task 5:

Among the following rectangles, circle the one you think is the most attractive and well balanced:



Give the reasons of your choice.

Now, measure each rectangle's length and width, and compare the ratio of length to width for each rectangle:

Rectangle	**1**	**2**	**3**
Length (L) cm			
Width (w) cm			
$\frac{L}{w}$			

Task 6 :

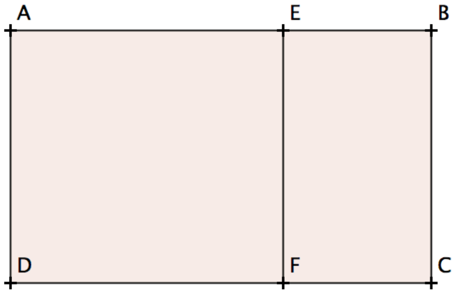
Find at least three rectangles in the room. Measure their dimensions. Fill in the following table :

Rectangle			
Length (L) cm			
Width (w) cm			
$\frac{L}{w}$			

Which one seems to be the most attractive?

Task 7:

We consider a rectangle ABCD of **length** AB=x (given $1<x<2$) and **width** AD=1. Let E be the point of [AB] and F the point of [DC] such that AEFD is a square



For each rectangle (ABCD and BEFC), find the ratio of the length to the width in terms of x :

r_{ABCD}=

r_{BEFC}=

ABCD is said to be a **golden rectangle** if $\mathbf{r_{ABCD}=r_{BEFC}}$.

Write the corresponding equation in terms of x and solve it.

One of the greater solution of this equation is :

ϕ=

This number is called the golden ratio or divine proportion or golden mean.

Give an approximate value for ϕ. (Round off to the nearest thousandths).

$\phi \cong$

Task 8 (première S) :

1) Find ϕ^2 and check that $\phi^2=\phi+1$.
2) From the last equality, deduce $\phi = 1 + \frac{1}{\phi}$.

Task 9: Write a presentation about the use of the golden ratio in Architecture.

Task 10: Write a presentation about the use of the golden ratio in Arts.

Task 11: Write a presentation about the use of the golden ratio in Nature.

Task 12: Being stuck on a Architect …

You're an architect and you have to draw the planes of a construction. As you're definitely a great artist, you decide to use the magnificent and unforgettable GOLDEN RECTANGLE!

But can you draw it? Give a protocol of construction.

Task 13: Nature of ϕ.

Assume that ϕ is a rational number, meaning that there exists 2 whole numbers a and b so that

$\phi=\frac{a}{b}$. (The fraction is shortened as much as possible).

1) We recall that an even number is of the form 2k and an odd number is of the form 2k+1

   (k : whole number). Can a and b be both even numbers?

2)
   2a) Given that $\phi^2 = \phi + 1$, show that $a^2 = b^2+ab$.
   2b) Deduce that $a^2-b^2=ab$.
3) If a and b are both odd numbers, what about ab, a^2, b^2, a^2-b^2? Can we have $a^2-b^2=ab$?
4)
   4a) Assume that a is even and b is odd and write the proof using the same pattern.
   4b) Assume that a is odd and b is even and write the proof using the same pattern.
5) Is ϕ a rational number?