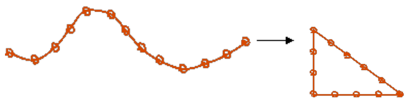
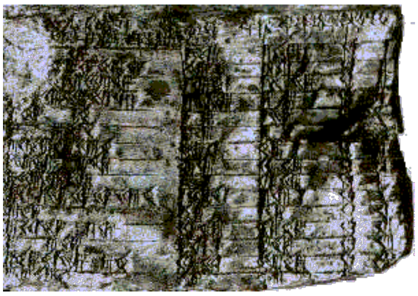
PYTHAGORAS' THEOREM

How did the Ancient Egyptians use the Pythagorean theorem?



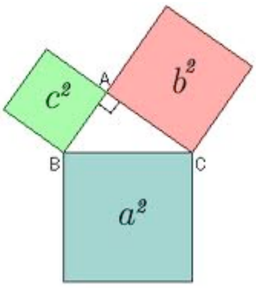
What for?



What about the Babylonians?

What is Greeks' greatest innovation in Mathematics?

What is the principle of **PROOF** in Mathematics?

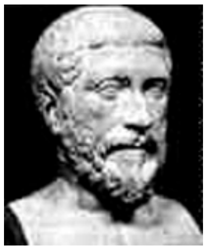


Write Pythagoras' theorem.

What is it useful for?

EXERCISES

Task 1



Who was Pythagoras? Write 5/10 lines about him and the Pythagorean School.

Task 2

MNP is a **right triangle at** P such that PM = 5 and PN = 8.

1. Draw a diagram.
2. Find the leng**th** MN.

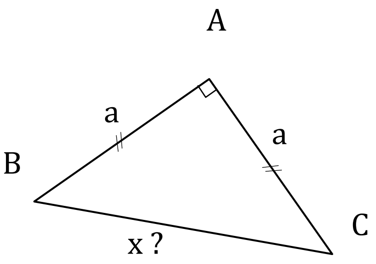
Task 3

ABCD is a **square** of side AB=BC=CD=AD=**a.** Find the length of the **diagonal** [AC] in terms of a.

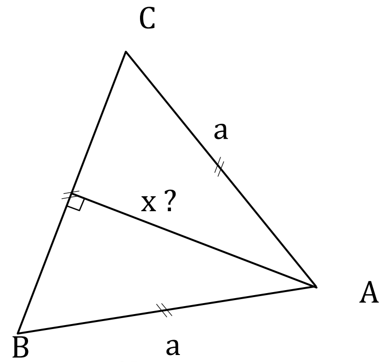
Task 4

The length **a** is given. Find the value of **x in terms of a** in each case:

•ABC is **a right triangle isosceles at** A

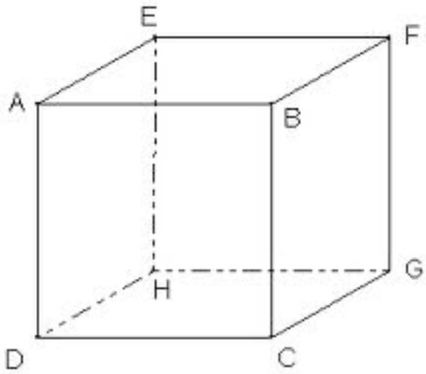


•ABC is an **equilateral** triangle



Task 5

ABCDEFGH is a cube of **edge** AB=a. Find the length of the **diagonal** [EC] in terms of a.



Task 6

A **Pythagorean Triple** is a set {a, b, c} of three **whole numbers** such that **$a^2+b^2 = c^2$.** A common example is the knotted rope of the Ancient Egyptians:

{3, 4, 5}.

1. Find some other Pythagorean triples.
2. Who were probably the first that used Pythagorean Triples?
3. Show that **IF** {a, b, c} is a Pythagorean triple, **THEN** so is {ka, kb, kc}, for k = 1, 2, …

THE PYTHAGOREAN SCHOOL

What is Pythagoras' main bringing-in compared with the Babylonians and Egyptians?

Where and when did Pythagoras found his school?

What were the main rules of this school?

Apart from the Pythagorean theorem, give two important discoveries involving the Pythagorean School.

EXERCISES

Task7

The Pythagoreans drew a simple square, with **sides equal to one**, and drew a diagonal.

1. Build such a figure.

As we can build geometrically this number, its actually exists. However, at that time, the Greeks only knew whole numbers, integers and rational numbers.

So the Pythagoreans' purpose was to write this number as the ratio of two integers to find its value.

But to their shock and horror, that number couldn't be rational.

2. What happened to Hippasus? Why?
3. **Proof of the Irrationality of** $\sqrt{2}$ (Proof **ad absurdum** or **by contradiction: assume** the opposite and show that it is false)

We **assume** that $\sqrt{2}$ **is a rational number**, meaning that there exists 2 whole numbers a and b so that $\frac{a}{b} = \sqrt{2}$. (the fraction $\frac{a}{b}$ is shortened as much as possible : it is reduced to the lowest term).

   a. **Check** that $a^2 = 2b^2$.

   b. Deduce that **a** is an even number

So we can write **a = 2p** (p: whole number).

   c. Prove that $2p^2 = b^2$. Deduce that b is an even number.

   d. Complete:

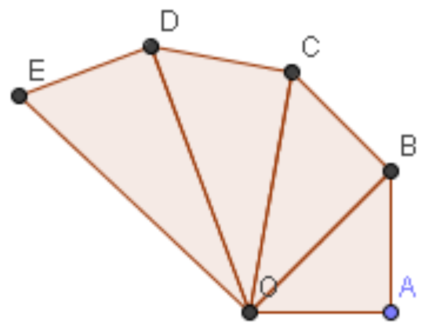
Thus a and b are both ....numbers, although $\frac{a}{b}$ is reduced to the lowest term! What a contradiction!

Conclusion: $\sqrt{2}$ is ....

EXERCISES

Task 8: a square root spiral

Square Root Spirals consist of right triangles, as shown in the figure below (OAB at A, OBC at B …, in which each outer segment ([AB], [BC], [CD] etc.) and the horizontal segment [OA] near the centre are of equal length 1.

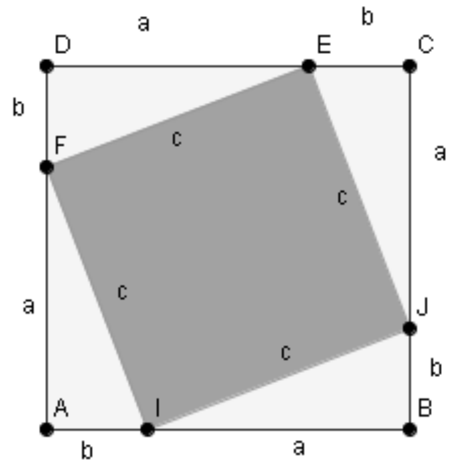


1. Find the lengths of the hypotenuses of each triangle (OB, OC, OD …).
2. Explain how you would build a segment of length $\sqrt{11}$.
3. Build 10 more triangles to complete the spiral.

Task 9: a proof of the Pythagorean theorem (among the hundreds of proofs that exist!)

On the given diagram, ABCD is a square.

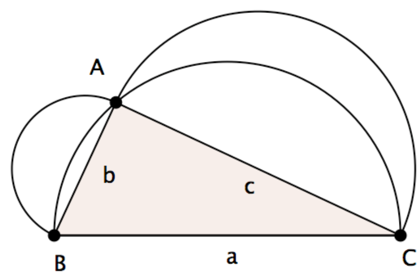
1. To show that EFIJ is a square, prove that it has 4 equal lengths and at least one right angle.
2. Find the expression area of ABCD in terms of a and b.
3. Find the expression of the area of each triangle in terms of a and b.
4. Deduce that $a^2+b^2=c^2$.



Task 10

Write the **<u>con</u>verse** of the Pythagorean theorem. Draw a figure.

Task 11 Hipopocrates lunes



$\mathcal{C}$ is a semi-circle of diameter [BC] outside the triangle. A is a point of $\mathcal{C}$. The length BC is denoted a.

Likewise, $\mathcal{C}_1$ is the semi-circle of diameter [AB]. The length AB is denoted c and $\mathcal{C}_2$ is the semi-circle of diameter [AC]. The length AC is denoted b.

1. Colour the crescent shapes.
2. Show that ABC is a right triangle.
3. Compare the area of the triangle ABC with the total area of the two crescent shapes.