

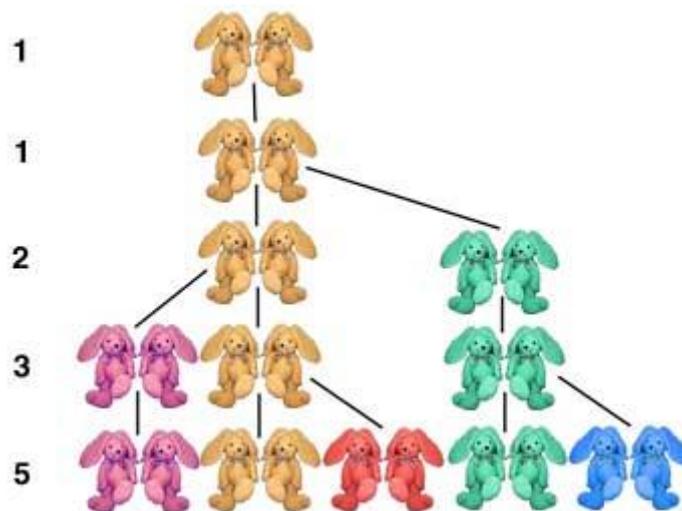
## The GOLDEN RATIO and the FIBONACCI SEQUENCE

In 1202, the Italian mathematician Leonardo da Pisa published the *Liber Abaci* – “the book of calculation”-, that promoted the use of Hindu-Arabic numerals and some financial computations.



One of the problems described in this book is about rabbits, and it goes like this: “A man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits are produced from that pair in a year, if it is supposed that every month each pair produces a new pair, which from the second month onwards becomes productive?”

A pair is mature if it can produce offspring, and immature if not. Step by step, we obtain the sequence 1, 1, 2, 3, 5, 8, 13, 21, 34 ... for months 1, 2, 3, 4, 5, 6, 7, 8 ...



By the XVIIIth century, Leonardo was given the nickname of Fibonacci – son of Bonaccio, and the list of numbers described above soon became known as the Fibonacci sequence. The symbol for the nth number of the sequence is  $F_n$ , starting with  $F_0=0$ .

The Fibonacci numbers have strong connection with the golden number  $\phi$ . If you take the ratio of two consecutive Fibonacci numbers, then the values of these ratios get closer and closer to  $\phi$ . In other words, the golden ratio is approximated by the ratio of two consecutive numbers, with the approximation increasing in accuracy further down the sequence.

### The GOLDEN RATIO and the FIBONACCI SEQUENCE ST A

1. What are Fibonacci Numbers? Write the first ten of them.
2. Explain how these numbers are found, i.e. the pattern to construct the sequence.
3. Who was Fibonacci? Find out:
  - a. his nationality
  - b. the period he lived,
  - c. his real name, and why people call him "Fibonacci",
  - d. Some other mathematics discoveries apart from the Fibonacci numbers.
4. Why is the list of numbers referred above known as the "Fibonacci sequence"?
5. Explain the problem of the Fibonacci rabbits:
  - a. What is the principle of their reproductive pattern?
  - b. Draw a diagram that illustrates this pattern.
  - c. Work out the link between the problem of the rabbits and the sequence.
6. Fill in the following table (use a spreadsheet software):

n	0	1	2	3	4	5			
$F_n$	$F_0$	$F_1$	$F_2$						
Value	0	1	1						

7. Work out  $F_{12}$ . What is the answer to the problem posed by Fibonacci?
8. Find  $F_{100}$ . Give an interpretation to this number.
9. Use the spreadsheet to compute the ratios of two consecutive terms of this sequence. How is this ratio moving as n increases? What is the name of the number the ratio tends to?

### The GOLDEN RATIO and the FIBONACCI SEQUENCE ST B

1. What are Fibonacci Numbers? Write the first ten of them.
2. Explain how these numbers are found, i.e. the pattern to construct the sequence.
3. Who was Fibonacci? Find out:
  - a. his nationality and his place of birth,
  - b. the period he lived,
  - c. his real name, and why people call him "Fibonacci",
  - d. Some other mathematics discoveries apart from the Fibonacci numbers.
4. Why is the list of numbers referred above known as the "Fibonacci sequence"?
5. Explain the problem of the Fibonacci rabbits:
  - a. What is the principle of their reproductive pattern?
  - b. Draw a diagram that illustrates this pattern.
  - c. Work out the link between the problem of the rabbits and the sequence.
6. Fill in the following table (program your calculator):

n	0	1	2	3	4	5			
$F_n$	$F_0$	$F_1$	$F_2$						
Value	0	1	1						

7. Work out  $F_{12}$ . What is the answer to the problem posed by Fibonacci?
8. Find  $F_{100}$ . Give an interpretation to this number.
9. Write a program to compute the ratios of two consecutive terms of this sequence. How is this ratio moving as n increases? What is the name of the number the ratio tends to?