# THE BISECTION METHOD

**preliminary exercise**

Let $f(x) = x^3 - 2x^2 + 3x - 4$.

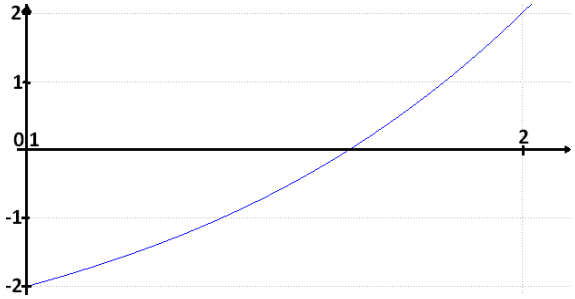
**1)** Find a root of the equation $f(x) = 0$ without using the calculator. (Hint: Good luck!)

**2)** Graph the equation $y = f(x)$ on your calculator for $x \in [0\ ,4]$.
Sketch the graph of $f$:

We deduce that there is one solution in the interval $[0; 4]$.
Give a first approximation of the solution of the equation above.

**3)** **The bisection method**

**3)a)** Calculate $f(1)$ and $f(2)$.




As expected $f(1) < 0$ and $f(2) > 0$, therefore, since $f$ is continuous, by the Intermediate Value Theorem, $f$ has a zero in $[1\ ,2]$.

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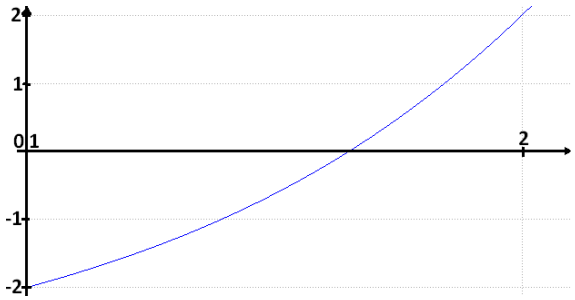
**3)b)** Now, calculate $f(1.5)$.




$f(1.5) < 0$, so there must be a solution between 1.5 and 2 and you've narrowed down your search area to $[1.5\ ,2]$.

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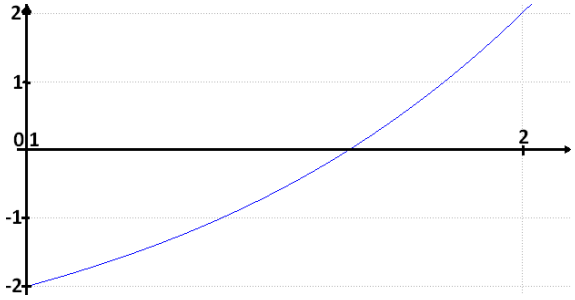
**3)c)** Now, calculate $f(1.75)$.

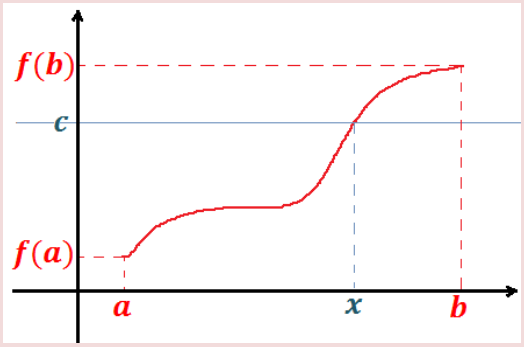



Give the sign of $f(1.75)$ and conclude.

## THE INTERMEDIATE VALUE THEOREM

Let $f: [a, b] \to \mathbb{R}$ be a continuous function, and $c$ be a real number.

$$\left| \begin{matrix} If\ f(a) < c < f(b) \\ or\ if\ f(a) > c > f(b) \end{matrix} \right.$$ then there exists an $x \in [a, b]$ such that $f(x) = c$.




*Note that the Intermediate Value Theorem doesn't say anything about how many times $f(x)$ takes the value $c$. There might be many values of $x$ in the interval $[a, b]$ such that $f(x) = c$. All the theorem says is that there is at least one.*