THE PYTHAGOREAN THEOREM

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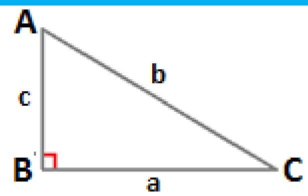
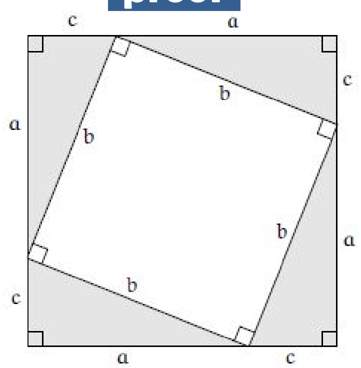
The Pythagorean Theorem

If ABC is a right-angled triangle with $\angle B$ the right angle, then $b^2 = c^2 + a^2$.



That means: "The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the lengths of the other two sides".

proof



$$\text{total area} = \text{area of the four triangles} + \text{area of middle square}$$

$$(a+c)^2 = 4\left(\frac{ac}{2}\right) + b^2$$

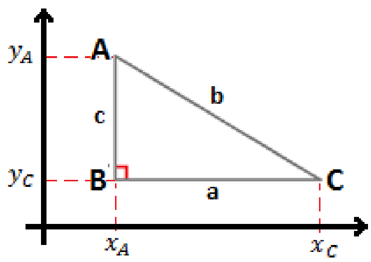
$$a^2 + 2ac + c^2 = 2ac + b^2$$

$$\boxed{a^2 + c^2 = b^2}$$

applications

* The discovery of the Pythagorean Theorem led the Greeks to prove the existence of numbers that could not be expressed as rational numbers, such as $\sqrt{2}$.

* The Pythagorean Theorem is used in calculating the distance between two points in both two and three dimensional space.



Suppose that $A(x_A, y_A)$ and $C(x_C, y_C)$ are two points in the plane.
Consider the right-angled triangle ABC where $B(x_A, y_C)$.
By the Pythagorean Theorem:

$$AC^2 = AB^2 + BC^2$$

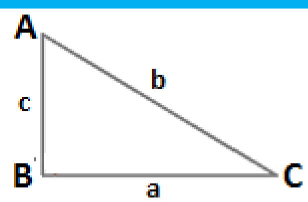
$$AC^2 = (y_C - y_A)^2 + (x_C - x_A)^2$$

Hence:

$$\boxed{AC = \sqrt{(y_C - y_A)^2 + (x_C - x_A)^2}}.$$

Converse of the Pythagorean Theorem

If $b^2 = c^2 + a^2$, then $\angle B$ is a right angle.



application

Given the lengths of the sides of a triangle, we can tell whether or not the triangle is right angled.