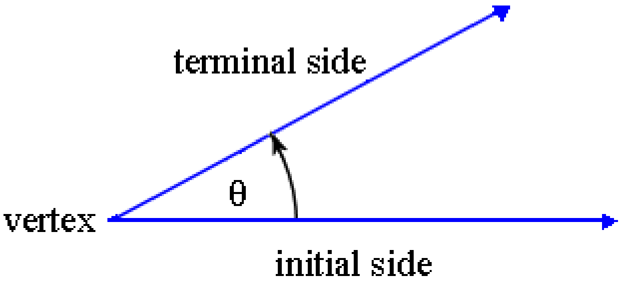
TRIGONOMETRY

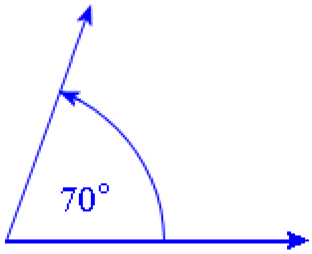
1. Angles



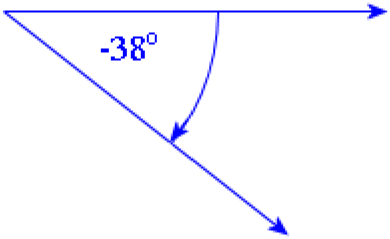
An **angle** is a measure of the amount of rotation between two line segments. The 2 line segments (or **rays**) are named the **initial side** and **terminal side** as shown in the diagram.

If the rotation is anti-clockwise, the angle is positive. Clockwise rotation gives a negative angle.

Examples



Anti-clockwise, positive angle.



Clockwise, negative angle.

Angles are commonly measured in degrees or radians.

(There is another unit for measuring angles, called **gradians**. The right angle is divided into 100 gradians. Gradians are used by surveyors, but not commonly used in mathematics. However, you will see a "grad" mode on most calculators.)

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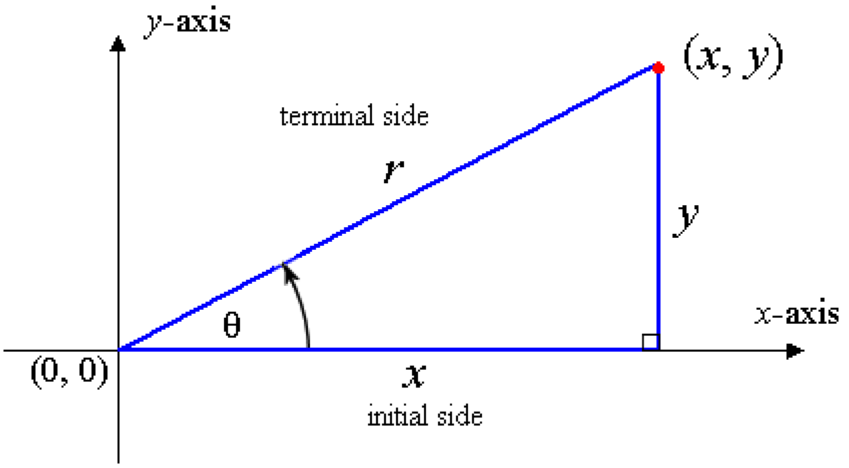
Standard Position of an Angle



An angle is in **standard position** if the initial side is the positive *x*-axis and the vertex is at the origin. The 2 examples given above are in standard position.

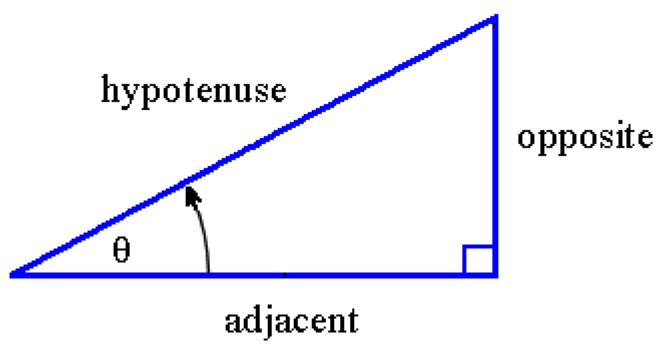
We will use *r*, the length of the hypotenuse, and the lengths *x* and *y* when defining the trigonometric ratios in the next section.

Degrees, Minutes and Seconds

The Babylonians (who lived in modern day Iraq from 5000 BC to 500 BC) used a base 60 system of numbers. From them we get the division of time, latitude & longitude and angles in multiples of 60.

Similar to the way hours, minutes and seconds are divided, the **degree** is divided into 60 minutes (') and a minute is divided into 60 seconds (").We can write this form as:DMS or ° ' ".

2. Sine, Cosine, Tangent.



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For the angle θ in a right-angled triangle as shown, we name the sides as:

- **hypotenuse** (the side opposite the right angle)
- **adjacent** (the side "next to" θ)
- **opposite** (the side furthest from the angle)

We **define** the three trigonometrical ratios **sine θ**, **cosine θ**, and **tangent θ** as follows (we normally write these in the shortened forms **sin θ**, **cos θ**, and **tan θ**):

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

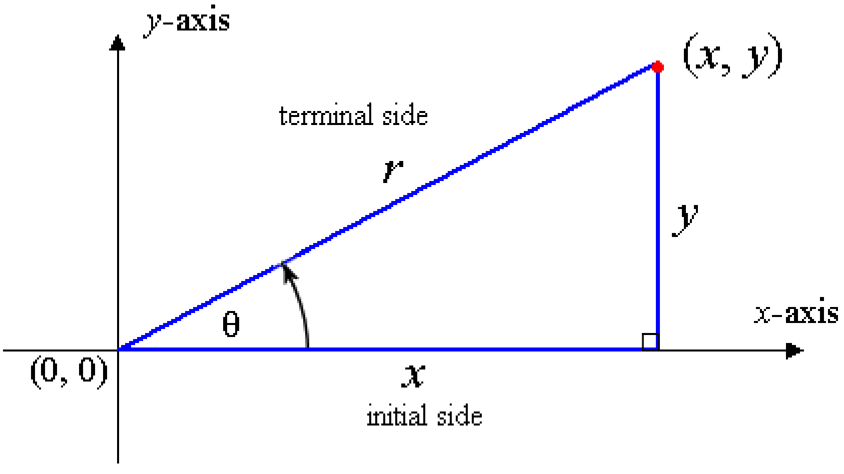
To remember these, many people use SOH CAH TOA, that is:

Sin θ = **O**pposite/**H**ypotenuse,

Cos θ = **A**djacent/**H**ypotenuse, and

Tan θ = **O**pposite/**A**djacent

The Trigonometric Functions on the *x*-*y* Plane



For an angle in standard position, we define the trigonometric ratios in terms of *x*, *y* and *r*:

$$\sin\theta = \frac{y}{r} \quad \cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}$$

Notice that we are still defining

sin θ as opp/hyp; cosθ as adj/hyp, and tan θ as opp/adj,

but we are using the specific *x*-, *y*- and *r*-values defined by the point (*x*, *y*) that the terminal side passes through. We can choose any point on that line, of course, to define our ratios.

To find *r*, we use Pythagoras' Theorem, since we have a right angled triangle:

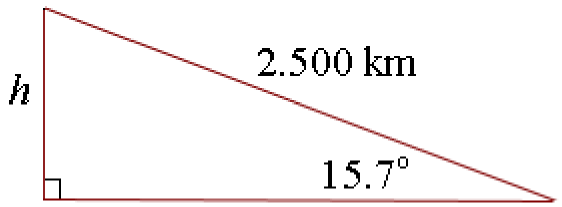
$$r = \sqrt{x^2 + y^2}$$

3. The Right Triangle and Applications

Many problems involve right triangles. We often need to use the trigonometric ratios to solve such problems.

Example 1 - Finding the Height

Find *h*.



Answer :

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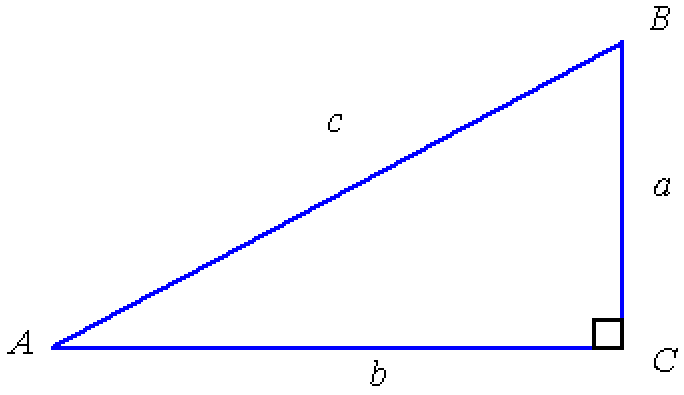
Example 2 - Solving Triangles

Solve the triangle ABC, given that A = 35° and c = 15.67.

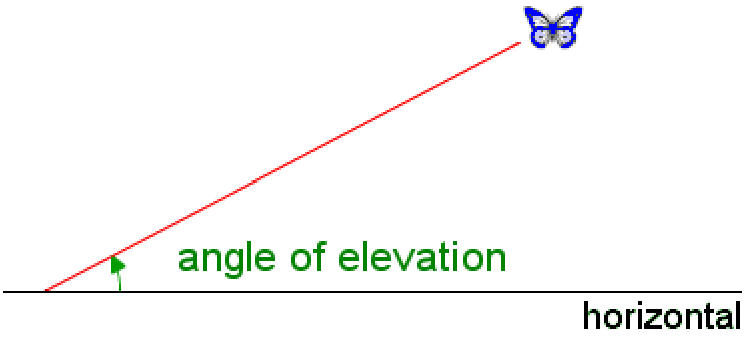


Answer :

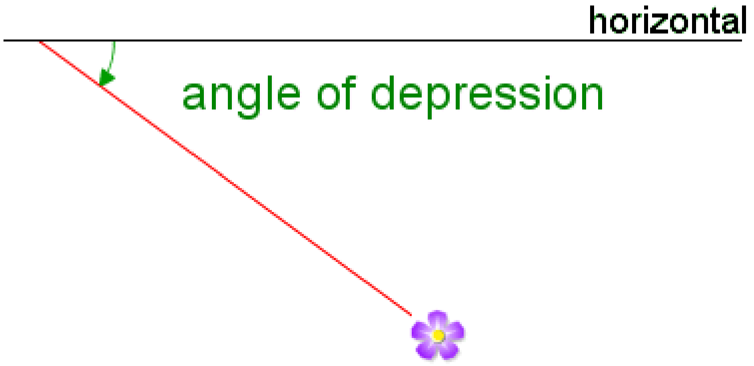
> *To "solve" a triangle means to find the unknown sides and angles. In this example, we need to find a and b and angle* B. *Note C = 90°.*

Angles of Elevation and Depression

In surveying, the **angle of elevation** is the angle from the horizontal looking **up** to some object:

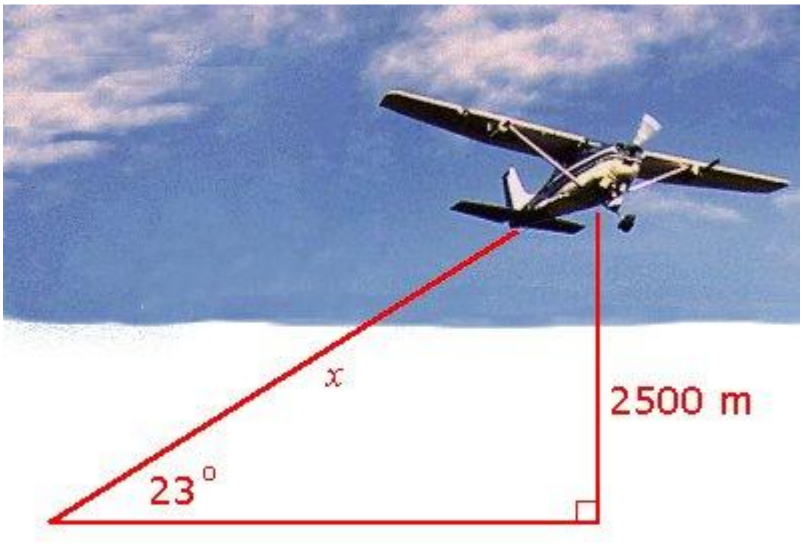


The **angle of depression** is the angle from the horizontal looking **down** to some object:



Example 3:

The angle of elevation of an aeroplane is 23°. If the aeroplane's altitude is 2500 m, how far away is it?



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Answer :

Example 4

You can walk across the Sydney Harbour Bridge and take a photo of the Opera House from about the same height as top of the highest sail.

This photo was taken from a point about 500 m horizontally from the Opera House and we observe the waterline below the highest sail as having an angle of depression of 8°. How high above sea level is the highest sail of the Opera House?

Answer :